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Monopoles and dyons in non-commutative geometry

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Abstract: Taking advantage of the equivalence between supersymmetric Yang-Mills theory on non-commutative spaces and the field theory limit of D3-branes in the background of NSNS 2-form field, we investigate the static properties of magnetic monopoles and dyons using brane construction techniques. When parallel D3-branes are separated by turning on a Higgs vacuum expectation value, D-strings will stretch between them at an angle which depends on the value of the background 2-form potential. These states preserve half of the supersymmetries and have the same masses as their commutative counterparts in the field theory limit. We also find stable (p, q)-dyons and string junctions.

Keywords: Solitons Monopoles and Instantons, D-branes, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Quantum field theories on non-commutative geometries have received renewed attention recently following the observation that they arise naturally as a decoupled limit of open string dynamics on D-branes [1]. In the formalism of [1], supersymmetric Yang-Mills theory on non-commutative geometry (NCSYM) arises from Fourier transforming the winding modes of D-branes living in a transverse torus in the presence of NSNS 2-form background [2]. To be concrete, consider a D-string oriented along the 01-plane and localized on a square torus in the 23-plane in the background of B_{23}. In the absence of B_{23}, the Fourier transform is equivalent to acting by T-duality in the 23-directions. In the presence of the B_{23}, however, the Fourier transform **(I)** and T-duality **(II)** acts differently. On one hand, **(I)** gives rise to the NCSYM with non-commutativity scale

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}\,. \tag{1.1}$$

On the other hand, **(II)** gives rise to D3-branes in the NSNS 2-form background. The precise map of degrees of freedom between **(I)** and **(II)** is highly non-local and was described in a recent paper [3] as a perturbative series in the non-commutativity parameter θ. The physics of **(I)** at large 't Hooft coupling can further be related to **(II)** in the near horizon region [4, 5] in the spirit of the AdS/CFT correspondence [6]. Yet, these equivalences have contributed very little to the understanding of the localized observables in the NCSYM. The difficulty stems largely from the fact that we do not yet understand the encoding of the observables in one formulation in terms of the other with sufficient detail.

To study the localized structures, it is natural to introduce localized probes. Topologically stable solution such as a magnetic monopole seems particularly suited for such a task. Instantons on non-commutative space-times have also been studied [7] along this line.

In this article, we will study the static properties of magnetic monopoles, dyons, and other related structures in the NCSYM with $\mathcal{N} = 4$ supersymmetry.[1] Since the non-commutativity modifies the equation of motion for the gauge fields, one must first establish the fact that these solutions exist in the first place. To this end, the equivalence between **(I)** and **(II)** will prove to be extremely useful; magnetic monopoles and dyons can be understood in **(II)** in the language of brane configurations. Masses, charges, and supersymmetries of these objects can be analyzed in the language of **(II)**. The fact that these objects stay in the spectrum of the theory in the decoupling limit provides a strong evidence that objects with the corresponding mass, charge, and supersymmetry exist in the NCSYM. In the language of **(II)**, it is also straightforward to argue for the existence and stability of exotic dyons which arise from three-string junctions [11, 12, 13, 14] and other complicated brane configurations.

This paper is organized as follows. We will begin in section 2 by briefly reviewing some basic facts about the NCSYM **(I)** and how they arise as a decoupling limit. Then we will take the magnetic monopole as a concrete example and study its static properties in the language of **(II)** in section 3. In section 4, we will describe how the analysis of section 3 can be generalized to (p, q)-dyons and string junctions. We will conclude in section 5.

2. Non-commutative Yang-Mills from string theory

In this section, we will review the string theory origin of the NCSYM. To be specific, let us take our space-time to have 3+1 dimensions. We will not consider the effect of making time non-commutative. Then, without loss of generality, we can restrict our attention to the case where the only non-vanishing component of the non-commutativity parameter is $\theta_{23} = -\theta_{32} = 2\pi\Delta^2$. ($\Delta$ has the dimension of length.) The NCSYM with coupling $\hat{g}_{\rm YM}$ and non-commutativity $\theta_{\mu\nu}$ is defined by the action

$$S = \mathrm{Tr} \int d^x 4 \left(\frac{1}{4\hat{g}_{\rm YM}^2} \hat{F}_{\mu\nu} * \hat{F}^{\mu\nu} + \cdots \right), \tag{2.1}$$

[1] Related 1/2-BPS and 1/4-BPS constant field-strength solutions on tori were discussed in [8, 9, 10].

where "$\cdots$" corresponds to the scalar and the fermion terms, $\hat{F}$ is the covariant field strength

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + \hat{A}_\mu * \hat{A}_\nu - \hat{A}_\nu * \hat{A}_\mu \,, \tag{2.2}$$

and the $*$-product is defined by

$$f(x) * g(x) = e^{i\frac{\theta_{\mu\nu}}{2}\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x'_\nu}} f(x)g(x')\bigg|_{x=x'} \,. \tag{2.3}$$

Relevant details about non-commutative geometry and the NCSYM are reviewed in [1, 3].

According to the construction of [1], this theory is equivalent to D3-branes in the background NSNS 2-form in the $\alpha' \to 0$ limit while scaling

$$g_{\mathrm{s}} = \frac{1}{2\pi}\hat{g}_{\mathrm{YM}}^2\sqrt{\frac{\alpha'^2}{\alpha'^2+\Delta^4}}\,, \qquad V_{23} = \Sigma_B^2 = \frac{\alpha'^2}{\alpha'^2+\Delta^4}\Sigma^2\,, \qquad B_{23} = \frac{\Delta^2}{\alpha'}\,, \tag{2.4}$$

and keeping Δ, Σ and $\hat{g}_{\mathrm{YM}}$ fixed. In the presence of D-branes, longitudinally polarized constant NSNS 2-form is not a pure gauge and has the effect of inducing a magnetic flux on the world volume. The magnetic fluxes in this context can be interpreted as the non-threshold bound state of D-strings oriented along the 1-direction. When multiple parallel D3-branes are present, the same number of D-strings get induced on each of the D3-branes. When the 23-directions is compactified on a torus of size $\Sigma_B = \alpha'\Sigma/\Delta^2$, the ratio of the number of induced D-strings and the number of D3-branes is precisely $n_1/n_3 = \Sigma^2/\Delta^2$.

The map between gauge fields $\hat{A}_\mu$ of the NCSYM **(I)** and the gauge fields A_μ living on the D1-D3 bound state **(II)** was constructed in [3] to leading non-trivial order in θ, and takes the form

$$\hat{A}_i = A_i - \frac{1}{4}\theta^{kl}\{A_k, \partial_l A_i + F_{li}\} + \mathcal{O}(\theta^2)\,. \tag{2.5}$$

The resummation of this series is not well understood at the present time.[2]

3. Magnetic monopoles in NCSYM

In this paper, we will study a variety of dyonic states in the NCSYM. It will however be convenient to first study the case of the BPS monopole as a prototype. The analysis for other cases will follow a similar pattern.

[2]The higher order corrections to (2.5) were studied recently in [15].

3.1 Basic notions of the NCSYM monopoles

We are interested in studying the properties of the monopole-like objects in the NCSYM **(I)**. To simplify our discussions, we will take our gauge group to be SU(2). Some basic properties of the NCSYM action is already manifest. First, the $*$-product acts like an ordinary product for the constant fields in the Cartan subalgebra of the gauge group. Therefore, NCSYM can be higgsed just like the ordinary SYM. This is important since BPS monopoles exist as a stable state in the higgsed SYM. Second, if we assume that only the magnetic field and one component of the scalar (say $\hat{\Phi}_9$) is non-zero, the terms in the action can be assembled into the form

$$S = \frac{1}{4\hat{g}_{\rm YM}^2}{\rm Tr}\int d^x 4 \left[\epsilon^{ijk}\left(\hat{F}_{ij} * D_k\hat{\Phi} + D_k\hat{\Phi} * \hat{F}_{ij}\right) + (\hat{F}_{ij} - \epsilon_{ij}{}^k D_k\hat{\Phi}) * (\hat{F}^{ij} - \epsilon^{ijk} D_k\hat{\Phi})\right]. \tag{3.1}$$

The second term in the integral is positive definite, so the action is bounded below by

$$S \geq \frac{1}{4\hat{g}_{\rm YM}^2}{\rm Tr}\int d^x 4\, \epsilon^{ijk}\left(\hat{F}_{ij} * D_k\hat{\Phi} + D_k\hat{\Phi} * \hat{F}_{ij}\right) = \frac{1}{2\hat{g}_{\rm YM}^2}{\rm Tr}\int d^x 4\, \partial_k \epsilon^{ijk}\left(\hat{F}_{ij} * \hat{\Phi}\right). \tag{3.2}$$

Thus the notion of the BPS bound exists also in the non-commutative theory.

Now, by definition, a magnetic monopole solution should have the property that

$$\hat{\Phi} \to \frac{U}{2}\sigma^3 \tag{3.3}$$

at large r, so the bound on the action can be made to take the form

$$S = \frac{U}{4\hat{g}_{\rm YM}^2}{\rm Tr}\int_{S_2} dS_k\, \epsilon^{ijk}\hat{F}_{ij}\sigma^3 . \tag{3.4}$$

Furthermore, in order for the action to be finite, F_{ij} should decay according to

$$\hat{B}^k = \frac{1}{2}\epsilon^{ijk}\hat{F}_{ij} = \frac{x^k\sigma^3}{2r^3}Q \tag{3.5}$$

at sufficiently large r where the system looks spherically symmetric. Therefore, (3.4) is evaluated as

$$S = \frac{2\pi Q}{\hat{g}_{\rm YM}^2}U . \tag{3.6}$$

In commutative theories, Q takes on integer values due to the Dirac's quantization condition. It is an important question whether there are corrections to Q in powers of (ΔU) for the non-commutative theory. Even in the non-commutative theory, however, the fields are slowly varying for large enough r, so we expect the standard commutative gauge invariance argument to hold. Therefore, we are lead to conclude that the magnetic monopoles of NCSYM have the same masses and charges as their commutative counterparts.

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Here we have argued in general terms that a self-dual magnetic monopole solution will saturate the BPS bound and has the same mass and the charge as in the commutative theory, provided that they exist. Unfortunately, the field equations of the non-commutative theory contain an infinite series of higher derivative interactions, making the task of proving the existence, as well as studying the detailed structure of these solutions, a serious challenge. However, even without the detailed understanding of magnetic monopole solutions in NCSYM, the equivalence between **(I)** and **(II)** can be exploited to establish some basic properties of these objects. For example, the existence, the stability, the mass, and the supersymmetry of these states can be understood in the language of brane construction in **(II)**. In this formalism, it is also easy to establish similar properties of (p, q)-dyons and string junctions. These brane constructions provide a strong evidence that the corresponding objects exist in **(I)**.

3.2 Brane construction of the NCSYM monopoles

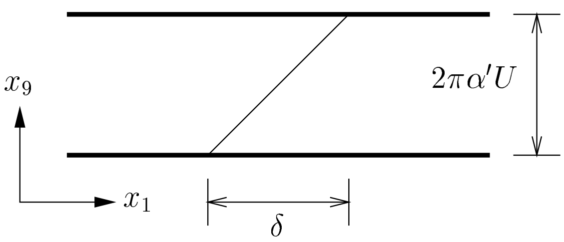

Figure 1: Brane configuration of a D-string suspended between a pair of parallel D3-branes in the background of the constant NSNS 2-form **(II)**. The induced magnetic field on the D3-brane world volume gives rise to a tilt in the D-string orientation.

In the formalism of the field theory brane constructions, magnetic monopoles in higgsed SYM have a natural realization as D-strings suspended between a pair of parallel but separated D3-branes. Similar configuration exists in **(II)** and is a natural candidate for a state which gets mapped to the magnetic monopole of **(I)** under the relation (2.5). One important difference between **(II)** and the usual situation is the fact that the background NSNS 2-form B_{23} also induces a background RR 2-form $A_{01} = \frac{1}{g}\sqrt{B_{23}^2/(1+B_{23}^2)}$ which couples to the world volume of the suspended D-string [5, 16]. This effect can also be interpreted as the force felt by the magnetic charge at the endpoint of the suspended D-string in the background of constant magnetic field in the 1-direction. The overall effect is to tilt the suspended D-string in the 1-direction and to change the overall energy of the configuration (see figure 1). The extent of the tilt and the change in the energy can be found by obtaining the minimal energy configuration of the D-string DBI action in the RR 2-form background at weak string coupling

$$S = \frac{1}{2\pi\alpha'} \int_0^{2\pi\alpha' U} dx_9 \left(\frac{1}{g_s} \sqrt{1 + \left(\frac{dx_1}{dx_9}\right)^2} + A_{01} \frac{dx_1}{dx_9} \right). \tag{3.7}$$

It is an elementary exercise to show that this expression is minimized for $dx_1/dx_9 = B$, and that the minimum mass is

$$m = \frac{U}{g_{\rm s}}\left(\sqrt{1+B^2} - \frac{B^2}{\sqrt{1+B^2}}\right) = \frac{2\pi}{\hat{g}_{\rm YM}^2}U\,, \tag{3.8}$$

where we used (2.4) to express the result in terms of the parameters of the NCSYM **(I)**. Despite the fact that the suspended D-string was tilted in the 1-direction in response to the background fields, the mass remained exactly the same as in the ordinary SYM.

It is also interesting to compute the "non-locality" of the suspended D-string indicated by "δ" in figure 1:

$$\delta = \frac{dx_1}{dx_9}2\pi\alpha' U = 2\pi\Delta^2 U\,. \tag{3.9}$$

This length therefore remains constant in the decoupling limit $\alpha' \to 0$ in spite of the fact that the slope dx_1/dx_9 diverges in this limit.

It is straightforward to count the number of supersymmetries preserved by this configuration. Let us denote the spinors representing 32 supercharges of type IIB theory by

$$\epsilon_- = \epsilon_L - \epsilon_R\,, \qquad \epsilon_+ = \epsilon_L + \epsilon_R\,. \tag{3.10}$$

As we mentioned earlier, D3-branes in the background of B_{23} can be thought of as a bound state of n_1 D-strings and n_3 D3-branes. Such a configuration places a constraint

$$\epsilon_- = \Gamma^0\Gamma^1(\sin(\phi)\epsilon_- + \Gamma^2\Gamma^3\cos(\phi)\epsilon_+) \tag{3.11}$$

on the supercharges, where $\tan(\phi) = B$. This result can be easily obtained by following the supersymmetry of $(p, q) = (n_1, n_3)$ string through a chain of duality transformations. On the other hand, a D-string tilted in the 19-plane by the angle $\phi = \tan^{-1}(B)$ preserves

$$\epsilon_- = \Gamma^0\Gamma^\phi\epsilon_-\,, \qquad \epsilon_+ = -\Gamma^0\Gamma^\phi\epsilon_+\,, \tag{3.12}$$

where

$$\Gamma^\phi = \Gamma^1\sin(\phi) + \Gamma^9\cos(\phi)\,. \tag{3.13}$$

The two constraints in (3.12) reduces the number of preserved supersymmetries from 32 to 16. It turns out that (3.11) closes among spinors satisfying (3.12), and reduce the number of independent supersymmetries from 16 to 8. Therefore, this brane configuration preserves the same number of supersymmetries as the magnetic monopole of $\mathcal{N} = 4$ SYM.

We are interested in the supersymmetry of these states in the field theory limit where we scale $B = \Delta^2/\alpha' \to \infty$ keeping Δ fixed. In this limit linear combinations of (3.11) and (3.12) can be assembled into the following independent set of conditions

$$\epsilon_- = \Gamma^0\Gamma^1\epsilon_- \,, \qquad \epsilon_+ = -\Gamma^0\Gamma^1\epsilon_+ \,, \tag{3.14}$$

$$\epsilon_- = \Gamma^9\Gamma^1\Gamma^2\Gamma^3\epsilon_+ \,. \tag{3.15}$$

These conditions are satisfied by 8 spinor components, indicating that the magnetic monopole preserves 8 out of 16 supercharges in the field theory limit.

The brane configuration described in this section is precisely the S-dual of the configuration considered in [17], except for the fact that in [17], it was the D3-brane that was tilted instead of the D-string. The two description can be mapped from one to the other by simply rotating the entire system. Although rotating the branes seem like a trivial operation, it amounts to changing the static gauge condition in the language of DBI action. The fact that this makes implicit reference to the gravitational sector of the theory means that this is not a symmetry in the field theory limit. It is more like a duality transformation mapping equivalent physical system between two descriptions. Let us therefore refer to the tilted D3-brane description as **(III)**.

One particular advantage of **(III)** is the fact that the field configuration corresponding to this brane configuration is easily understood. Thinking of the pair of D3-branes as giving rise to $U(2) = U(1) \times SU(2)$ gauge theory, the configuration of figure 2 is simply the $F_{23} = \partial\Phi_9 = B$ embedded into the U(1) sector and an ordinary Prasad-Sommerfield monopole embedded into the SU(2) sector [18].

The equivalence between **(II)** and **(III)** also sheds light on the nature of **(II)** when expanded in θ. When **(III)** is interpreted as a BIon, the fields are well defined as a single valued function. When **(III)** is rotated to **(II)**, this single-valuedness is lost. The field configuration must now contain branch cuts to account for multi-valuedness in some region of the D3-brane world volume. Since such a field configuration is non-analytic, expansion in θ is likely not to yield a uniformly converging series, and this may have profound implication for the map between **(I)** and **(II)**. Especially in light of the fact that **(II)** seem pathological from many points of view, having a more conventional alternative description **(III)** may prove to be extremely useful in future investigations.

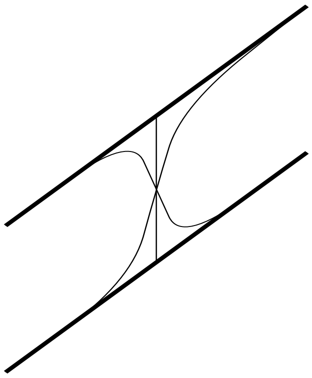

Figure 2: Magnetic monopole solution in the tilted D3-brane picture **(III)** where the world volume fields are single-valued.

3.3 Magnetic monopoles at large-N and large 't Hooft coupling

Before concluding this section, let us pause for a moment and briefly describe what happens to the magnetic monopoles in the NCSYM with large 't Hooft coupling and large N. Consider $SU(N+1)$ broken to $SU(N) \times U(1)$. At large coupling, this $SU(N)$

sector is described by the supergravity background [4, 5] and the U(1) sector appears as a D3-brane probe in this background. The supergravity background describing the near horizon of the N D3-branes in the background of B_{23} is given by

$$ds^2 = \alpha' \left\{ \left(\frac{U^2}{\sqrt{\lambda}}\right)(-dt^2 + dx_1^2) + \left(\frac{\sqrt{\lambda}U^2}{\lambda + \Delta^4 U^4}\right)(dx_2^2 + dx_3^2) + \frac{\sqrt{\lambda}}{U^2} dU^2 + \sqrt{\lambda} d\Omega^2 \right\},$$

$$e^{\phi} = \frac{\hat{g}_{\rm YM}^2}{2\pi}\sqrt{\frac{\lambda}{\lambda + \Delta^4 U^4}}, \qquad A_{01} = \frac{2\pi}{\hat{g}_{\rm YM}^2}\frac{\alpha'\Delta^2 U^4}{\lambda}, \qquad B_{23} = \frac{\alpha'\Delta^2 U^4}{\lambda + \Delta^4 U^4}, \tag{3.16}$$

where $\lambda = 4\pi \hat{g}_{\rm YM} N$. We wish to find the minimal configuration for the probe D-string action

$$S = \frac{1}{2\pi\alpha'}\int dx_1 \left(e^{-\phi}\sqrt{-G_{00}(G_{11} + G_{UU}(\partial U(x_1))^2)} - A_{01}\right). \tag{3.17}$$

Near the probe D3-brane, magnetic charge of the D-string will feel the same force as in the case of the flat space, so we impose the boundary condition that $\alpha'\partial U = \alpha'/\Delta^2$ at U where we place the probe D3-brane. Rather remarkably, the configuration

$$U(x_1) = \frac{1}{\Delta^2}x_1, \tag{3.18}$$

i.e. a tilted straight line, is a solution to this problem, and when the solution and the background is substituted into (3.17) we find

$$S = \int dx \frac{2\pi}{\hat{g}_{\rm YM}^2}\frac{1}{\Delta^2} = \frac{2\pi}{\hat{g}_{\rm YM}^2}U, \tag{3.19}$$

which, as expected for a BPS state, is the same mass that we found in the weakly coupled limit.

4. (p, q)-dyons and string junctions in NCSYM

In the previous section, we described the interpretation of magnetic monopoles of the NCSYM in the language of **(II)** and found that they have the same mass as the ordinary SYM. It is extremely straightforward to repeat the analysis of the previous section to the case of (p, q)-dyons. There will be some qualitative difference in the pattern of supersymmetry breaking which we will discuss below. Once the basic properties of the (p, q)-dyons are understood, it is natural to consider the possibility of forming a state corresponding to a string-junction [11, 12, 13, 14]. We will examine the existence, the stability, and the supersymmetry of these junction states.

4.1 (p, q)-dyons in NCSYM

It is extremely straightforward to generalize the discussion of the previous section to the (p, q)-dyon. The expression for the action (3.7) is generalized to

$$S = \frac{1}{2\pi\alpha'}\int_0^{2\pi\alpha' U} dx_9 \left(\sqrt{p^2 + \frac{q^2}{g_s^2}}\sqrt{1 + \left(\frac{dx_1}{dx_9}\right)^2} + qA_{01}\frac{dx_1}{dx_9}\right), \tag{4.1}$$

which is minimized by setting

$$\frac{dx_1}{dx_9} = \frac{qB}{\sqrt{(1+B^2)g_s^2p^2+q^2}}\,. \tag{4.2}$$

The minimum mass is

$$m = \sqrt{\frac{(1+B^2)g_s^2p^2+q^2}{(1+B^2)g_s^2}}U = \sqrt{p^2+\frac{4\pi^2q^2}{\hat{g}_{\rm YM}^2}}U\,, \tag{4.3}$$

which is precisely identical to the result one would expect from the ordinary SYM.

Let us now investigate the number of preserved supersymmetries for these dyons. For the sake of concreteness, we will first consider $(p,q)=(1,0)$, which is a W-boson. As in the previous section, the D3-brane puts the constraint (3.11). The $(1,0)$-string, on the other hand, preserves

$$\epsilon_- = \Gamma^0\Gamma^9\epsilon_+\,. \tag{4.4}$$

For spinors satisfying (4.4), the supersymmetry constraint (3.11) simplifies to

$$\Big(1-\Gamma^0\Gamma^1\sin(\phi)-\Gamma^1\Gamma^2\Gamma^3\Gamma^9\cos(\phi)\Big)\,\epsilon_- = 0. \tag{4.5}$$

Conditions (4.4) and (4.5) are satisfied by 8 independent spinor components for arbitrary values of ϕ.[3] Therefore, we learn that the W-boson in the field theory limit $\tan(\phi)=B=\Delta^2/\alpha'\to\infty$ also preserves 8 supercharges.

It is straight forward to extend this analysis to the case of (p,q) dyons. Taking the (p,q)-string to be oriented in the direction given by (4.2), it is easy to obtain a set of independent constraints in a manner similar to the monopole in the last section. The number of the unbroken supersymmetries is 8, and in the decoupling limit, the surviving supersymmetries are specified by (3.14) in addition to the constraint

$$\sqrt{p^2+\left(\frac{2\pi q}{\hat{g}_{\rm YM}^2}\right)^2}\,\Gamma^9\epsilon_- = \Gamma^1\left(p+\frac{2\pi q}{\hat{g}_{\rm YM}^2}\Gamma^2\Gamma^3\right)\epsilon_+\,, \tag{4.6}$$

which reduces to (3.15) when $(p,q)=(0,1)$. We conclude that in the field theory limit, the (p,q)-dyons are 1/2 BPS objects, precisely analogous to the situation in the ordinary $\mathcal{N}=4$ SYM.

Just as in the magnetic monopole case, one can consider the analogue of **(III)** where one tilts the D3-brane in such a way to make the (p,q)-string point upward. This will simply correspond to embedding the Julia-Zee dyon in the SU(2) sector and turning on the U(1) part independently. From this standpoint, it is easy to see that the number of preserved supersymmetries is 8.

[3] We thank M. Krogh for pointing out an error regarding this point in the earlier version of this paper.

The large N and large 't Hooft coupling limit of the (p,q)-dyon is also straightforward to analyze. One simply generalizes (3.17) to

$$S = \frac{1}{2\pi\alpha'} \int dx_1 \left(\sqrt{p^2 + q^2 e^{-2\phi}} \sqrt{-G_{00}(G_{11} + G_{UU}(\partial U(x_1))^2)} - qA_{01} \right). \qquad (4.7)$$

The minimal action configuration satisfying the appropriate boundary condition is simply

$$U(x) = \frac{x}{\Delta^2} \sqrt{1 + \frac{\hat{g}_{\rm YM}^4 p^2}{4\pi^2 q^2}}\,, \qquad (4.8)$$

and we find the mass of the (p,q)-dyon to be

$$m = U \sqrt{p^2 + \frac{4\pi^2 q^2}{\hat{g}_{\rm YM}^4}}, \qquad (4.9)$$

in agreement with the earlier result from weak coupling (4.3).

4.2 String junctions in NCSYM

Having established the existence and some basic properties of (p,q)-dyons, it is natural to consider the status of string junctions. In the absence of the background NSNS 2-form, the existence of string junction relied on the property of (p,q)-strings, that their tension can be balanced

$$\sum_i \vec{T}_{p_i,q_i} = 0\,, \qquad (4.10)$$

where

$$\vec{T}_{p,q} = \left(p, \frac{q}{g_{\rm s}}\right) \qquad (4.11)$$

for $\sum p_i = \sum q_i = 0$. The components of $\vec{T}$ can be, say, in the 8 and the 9 directions.

When the effect of the B-field is taken in to account, these vectors are rotated out of the 89-plane into the 1-direction. Now one needs to make sure that the tension balance condition is satisfied in the 1, 8, and 9 directions simultaneously. It turns out, however, that the entire effect of the B-field can be accounted for by rotating the tension vector in the 19-plane so that the (1,8,9) components read

$$\vec{T}_{p,q} = \left(\frac{q}{g_{\rm s}}\sin(\phi),\ p,\ \frac{q}{g_{\rm s}}\cos(\phi)\right), \qquad \tan(\phi) = B\,. \qquad (4.12)$$

It is straightforward to verify that this vector is oriented relative to the D3-brane world volume with the appropriate slope (4.2) by rotating $T_{p,q}$ in the 89-plane to point in the 19-directions.

Since we can just as easily tilt the D3-branes instead of tilting the (p,q)-strings, there is a version of **(III)** for the string junction. The fact that the field configuration for such a state is known [19, 20, 21, 22] might prove useful in the same way that the Prasad-Sommerfield solution in **(III)** is related to the magnetic monopole in the NCSYM **(I)**.

Clearly, the condition for sum of $\vec{T}_{p,q}$ to vanish for conserved (p,q)-charges in a string junction is still valid, so the string junction exists as a stable state in the presence of the B field. Though different supersymmetries are preserved by the respective component (p,q)-strings in the string network, in view of this stability the whole configuration is expected to preserve some of the supersymmetries. Let us therefore investigate the field theory limit of these configurations more closely.

Consider a junction of strings (p_i, q_i), $i = 1, 2, 3$, supported by D3-branes localized in the 89-plane with strings meeting at the origin. In order to take the field theory limit of such a configuration, we should scale the distance of the D3-brane to the origin as $\alpha' U_i$ with $\alpha' \to 0$ and oriented in the $(p, \frac{q}{g_s\sqrt{1+B^2}})$ direction in the 89-plane. In other words, the Higgs expectation value of the (Φ_8, Φ_9) field should be chosen to scale according to

$$\vec{U}_i = (\Phi_8, \Phi_9)_i = \frac{U_i}{\sqrt{p_i^2 + \frac{q_i^2}{(1+B^2)g_s^2}}} \left(p_i, \frac{q_i}{g_s\sqrt{(1+B^2)}} \right). \tag{4.13}$$

To take the field theory limit, we scale g_s and B according to (2.4). Expressed in terms of $\hat{g}_{\mathrm{YM}}$ and Δ, (4.13) reads

$$\vec{U}_i = (\Phi_8, \Phi_9)_i = \frac{U_i}{\sqrt{p_i^2 + \frac{4\pi^2}{\hat{g}_{\mathrm{YM}}^4} q_i^2}} \left(p_i, \frac{2\pi}{\hat{g}_{\mathrm{YM}}^2} q_i \right) \tag{4.14}$$

and has a trivial $\alpha' \to 0$ limit. These junction states therefore appear to exist in the field theory limit and orient itself in the usual way in the 89-plane as we illustrate in figure 3. Figure 3 does not represent the orientation of the strings outside the 89-plane but it should be remembered that they are tilted in the 19-plane. The mass of the junction takes the same form as in the commutative case

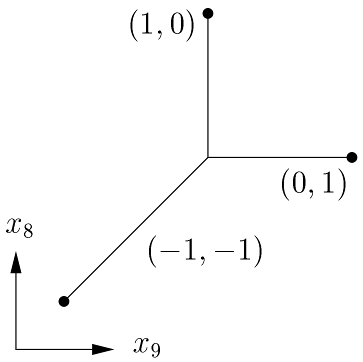

Figure 3: Configuration of three string junction in a NSNS 2-form background. The dots denote the D3-branes perpendicular to the 89-plane. The orientation of the branes resembles the conventional junction in the 89-plane. The components of the junction is tilted in the 19-plane in response to the NSNS 2-form background.

$$m = \sum_{i=1,2,3} \sqrt{p_i^2 + \frac{4\pi^2}{\hat{g}_{\mathrm{YM}}^4} q_i^2} |\vec{U}_{p_i,q_i}| \,. \tag{4.15}$$

The unbroken supersymmetries of the junction in the field theory limit corresponds to the spinor components of the supercharges satisfying the constraints of both the monopoles and the W-bosons, (3.14), (3.15), and (4.4). This can be seen easily from the fact that, since the (p_i, q_i)-string is now oriented in the direction

(4.14) in the 89-plane in the decoupling limit, the constraint for the component (p_i, q_i)-string becomes

$$\left(p_i\Gamma^8 + \frac{2\pi q_i}{\hat{g}_{\rm YM}^2}\Gamma^9\right)\epsilon_- = \Gamma^1\left(p_i + \frac{2\pi q_i}{\hat{g}_{\rm YM}^2}\Gamma^2\Gamma^3\right)\epsilon_+\,, \tag{4.16}$$

as a generalization of (4.6). We conclude, therefore, that objects in the NCSYM corresponding to the field theory limit of the string junctions preserves 4 supercharges, just like their commutative counterparts.

5. Conclusions

The goal of this paper was to understand the static properties of the magnetic monopole solution and its cousins in the NCSYM. Instead of working with the lagrangean formulation of NCSYM **(I)**, we took advantage of the equivalence between NCSYM **(I)** and the decoupling limit of D3-branes in a background NSNS 2-form potential **(II)** to study the stable brane configurations corresponding to these states. Using this approach, it is extremely easy to show that there are stable brane configurations corresponding to magnetic monopoles, (p, q)-dyons, and string junctions, and that they survive in the field theory limit.

Having established some basic properties of these objects in the language of brane construction, it is natural to wonder how much of this can be understood strictly in the frame work of the lagrangean formalism. It would be especially interesting to find an explicit solution which generalizes the standard Prasad-Sommerfield solution [23] to the non-commutative setup. It is very encouraging that the construction of instanton solutions via the ADHM method admits a natural non-commutative generalization [7]. Indeed, Nahm's construction of the magnetic monopole [24, 25, 26] also admits a simple non-commutative generalization. One simply solves for the normalized zero modes of the operator

$$0 = \hat{\mathbf{\Delta}}^\dagger * \hat{\mathbf{v}} = i\frac{d}{dz}\hat{\mathbf{v}}(z,x) - \mathbf{x} * \hat{\mathbf{v}}(z,x) - \mathbf{T}\hat{\mathbf{v}}(z,x) \tag{5.1}$$

and computes

$$\hat{A}_i = \int_{-U/2}^{U/2} dz\, \mathbf{v}^\dagger(z,x) * \partial_i \mathbf{v}(z,x)\,, \qquad \hat{\Phi} = \int_{-U/2}^{U/2} dz\, z\mathbf{v}^\dagger(z,x) * \mathbf{v}(z,x)\,. \tag{5.2}$$

The non-commutativity is reflected in the $*$-product in (5.1), and as long as $\mathbf{\Delta}^\dagger * \mathbf{\Delta}$ satisfies the usual requirement that it be invertible and that it commutes with the quarternions, all the steps in the argument leading to the self-duality of (5.2) follow immediately from the same argument in the commutative case [27, 28]. Despite tantalizing similarities with the commutative case, we were not able to solve (5.1) in closed form to proceed further. It would be very interesting to see if an explicit expression for the non-commutative BPS monopole can be found.

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